Bulgarian Journal of Agricultural Science, 19 (No 2) 2013, 383-386
Agricultural Academy

ON THE BIAXIAL DISTRIBUTION OF ANISOTROPIC ELASTIC MODULUS OF THIN-PLATE POLYCRYSTALLINE ELEMENTS AND HIS IDENTIFICATION IN AGRICULTURAL MACHINERY WITH ULTRASONIC METHODS

A. LEVI, H. BELOEV and N. KOLEV
University of Ruse 'A.Kanchev', BG – 8000 Ruse, Bulgaria

Abstract

LEVI, A., H. BELOEV and N. KOLEV, 2013. On the biaxial distribution of anisotropic elastic modulus of thin-plate polycrystalline elements and his identification in agricultural machinery with ultrasonic methods. *Bulg. J. Agric. Sci.,* 19: 383-386

A new method for identification of texture in agricultural machinery with interpolation of biaxial anisotropy of elastic modulus of thin-plate polycrystalline elements has been developed. The dependence of shear modulus and modulus of longitudinal elasticity as a function of an angular distribution of Poisson's ratio, achieved by the method of ulrasonic critical-angle refractometry (UCR), has been derived.

Key words: anisotropy of elastic modulus, thin-plate polycrystalline elements, ulrasonic critical-angle refractometry, ultrasonic non-destructive testing

Introduction

Agricultural machinery is made up of very diverse in shape and size structural elements. In their production details are subject to various processing operations, because of which significant residual stresses often appear. As they balance each other, their impact on polycrystal apriori structure can be determined by examining the emerging texture. Agricultural machines operate in a wide temperature range and are subjected to complex dynamic loads. Because of the texture, corrosion arises and the simultaneous action of external mechanical loading and the residual stress reduces the operational resource. This effect is particularly strong after the tempering of steel bars and tubes, cutting and heat treatment of various details. The study of biaxial texture with diffraction (radiographic) method is too inaccurate for technical metals are polycrystalline aggregates. On the other hand, the method of measuring the residual stresses by cutting the layers of the work piece surface and the measurement of deformation has occurred (and the method of trepanation) are too labor-intensive and impractical in production. In general, non-destructive methods for control of the mechanical biaxial texture and associated residual stresses are not yet sufficiently developed in the context of their application in the manufacture, repair and restoration of agricultural machinery.

According to the accepted classification (Ushio et al., 1993), a thin-plate is considered a construction element with thickness less than 3 mm.

A biaxial anisotropy appears in them, as the crystallographic plane is parallel to the plane of rolling, and the crystallographic axes depend on its direction (Randle and Engler, 2000). Generally the dependence of the averaged values of the stresses $\langle \sigma_{ij} \rangle$ and strains

E-mail: alevi@ecs.uni-ruse.bg

$\langle \varepsilon_{mn} \rangle$ in each point are defined by Hooke's law (Rychlewski, 1984):

$$\langle \sigma_{ij} \rangle = C^{eff}_{ijmn} \langle \varepsilon_{mn} \rangle \tag{1}$$

where: C^{eff}_{ijmn} - tensor of the effective modules of elasticity.

If polycrystalline aggregates are constructed of FCC and BCC crystals (Face and Body Centere Cubic Lattice), it's typical for Al, Au, Cu, Pb, Fe, W et al., in order to determine their deformation of elasticity the following three constants C_{11}, C_{12}, C_{44} (modules of elasticity of third-order) are necessary.

In hexagonal crystals: Mg, Zn,Cd et al. the constants are five: C_{11}, C_{12}, C_{44}, C_{33} and C_{13} (Jones and March, 1973; Kocks et al. (eds.), 1998).

As the prevailing majority of construction elements are from the first kind, anisotropy can be calculated using the parameter of Ziner - A_c.

Index of anisotropy $A_c = 2 / (\xi_x - \xi_y)$ depends on Cauchy relation for elastic constants: $\xi_x = C_{11} / C_{44}$ and $\xi_y = C_{12} / C_{44}$. Actually ξ_y is a measure of approximation to the model of Cauchy (at ξ_y=1 the strengths among the atoms do not depend upon the direction and $C_{11} = 3C_{12}$).

Basic Equations and Problem Formulation

According to Voigt (1928), the maximum value of the shear modulus $G^v = G_{max}$ in permanent deformation of the polycrystalline aggregates can be calculated with the following equation (2):

$$G^v / C_{44} = (2 / A_c + 3) / 5 \tag{2}$$

In the opinion of Reuss (Bunge, 1993) the minimum value of the same module $G^R = G_{min}$ (at constant tension exercised over the polycrystalline aggregates) can be calculated with the help of equation (3):

$$G^R / C_{44} = (2A_c + 3) / 5 \tag{3}$$

If $z_c = G_{max} / G_{min} = \left(C_T^{max} / C_T^{min}\right)^2$, when we assume that C_T is speed of propagation of transverse ultrasonic waves, then, following from (Lewi, 2010), we can define A_c:

$$A_c^2 - (25z_c - 13)A_c / 12 + 1 = 0 \tag{4}$$

The classical thermodynamic theory of solid bodies (Hill, 1986) gives the possibility to determine the characteristic value θ_D[K] (Debye temperature) in the next equation:

$$\theta_D = 0.00362\, \rho^{-1/6} M^{-1/3} (10K_0)^{1/2} [f(v)]^{-1/3}, \tag{5}$$

where: ρ - density of solid body, g / cm^3,
M_a - atomic mass in units,
K_0 - **modulus** of **volume elasticity,** GPa,
v - Poisson's ratio,

$$f(\nu) = \left[(1+\nu)/(3-3\nu)\right]^{3/2} + 2\left[(1+\nu)/(1.5-3\nu)\right]^{3/2}$$

- Debye function.

Solution of the Problem

In the formula of Lindemann (Hoffmann, 2004), Debye temperature θ_D[K] is connected to the melting temperature T_m[K] by the next equation:

$$\theta_D = k_0^* \sqrt{T_m \rho^{2/3} / M_a^{5/3}} \quad , \tag{6}$$

where: $k_0^* \approx 137$.

From (5) and (6) for the polycrystalline continuum, we get:

$$K_0 / \rho = k_0^{**} (T_m / M_a) [f(v)]^{2/3}, \tag{7}$$

where: k_0^{**} - material constant.

The even many-sided pressure does not cause a texture, so it is appropriate to express K_0 using G and v by the relation: $K_0 = G(1 + v) / (1.5 - 3v)$.

The parameters T_m, M_a and ρ do not depend or depend to a very small extent on the changes of the temperature, so we can define the biaxial anisotropy by the values of $\left[G_1(\varphi_1), \nu_1(\varphi_1)\right]$ and $\left[G_2(\varphi_2), \nu_2(\varphi_2)\right]$ in two optionally chosen directions, fixed with the polar angles φ_1 and φ_2.

Based on (7), we get the dependence:

$$\left[G_1(\varphi_1)/G_2(\varphi_2)\right]^{3/2} = \frac{2 + \left[(0.5-\nu_1)/(1-\nu_1)\right]^{3/2}}{2 + \left[(0.5-\nu_2)/(1-\nu_2)\right]^{3/2}} \tag{8}$$

Using the methods of ulrasonic critical-angle refractometry (UCR), and the immersion effects of leaky

Rayleigh or Lamb waves (Lewi, 2010; Śliwinski, 2000), we can find v_{min} and v_{max} of thin-plate polycrystalline elements. The Figure 1 presents the changes of reflection factor $\left|\dot{R}_L(\theta)\right|$ of ultrasonic wave with frequency 10MHz for thin-plate steel elements (UCR test : C_0 = 1.6 mm/μs).

They are presented as a function of refractive angle θ. Minimum $\left|\dot{R}_L(\theta)\right|$ corresponds to the third critical angle θ_R and $C_R = C_0 / \sin\theta_R$ is the group velocity of the leaky Rayleigh waves. For rotation in the plane orthogonal to the plane of wave propagation (φ is the angle of rotation) occur changes in the value of the critical angle $\Delta\theta_R$ (Figure 2). They are the result of mechanical anisotropy and occur in large plastic loads. With classical methods of ultrasonic measuring technique (Śliwinski, 2000) can determine C_L^{max}, C_L^{min} and the extremes of $v(C_R / C_L)$. In this case, from equation (8) we get:

$$z_c = \frac{G_{max}}{G_{min}} = \left\{ \frac{2 + \left[\left(0.5 - \nu_{min}\right)/\left(1 - \nu_{min}\right)\right]^{3/2}}{2 + \left[\left(0.5 - \nu_{max}\right)/\left(1 - \nu_{max}\right)\right]^{3/2}} \right\}^{2/3} \tag{9}$$

By defining the value of z_c the calculation of the parameter of Ziner (Lewi, 1988) from relation (10):

$$A_c = \left(25z_c - 13\right)/12 + \sqrt{\left(25z_c - 13\right)^2 / 144 - 1} \tag{10}$$

Here we have to say that the changes in the values of G and v are reciprocal, so as when $G \rightarrow G_{max}$ we get $v \rightarrow v_{min}$ and vice versa.

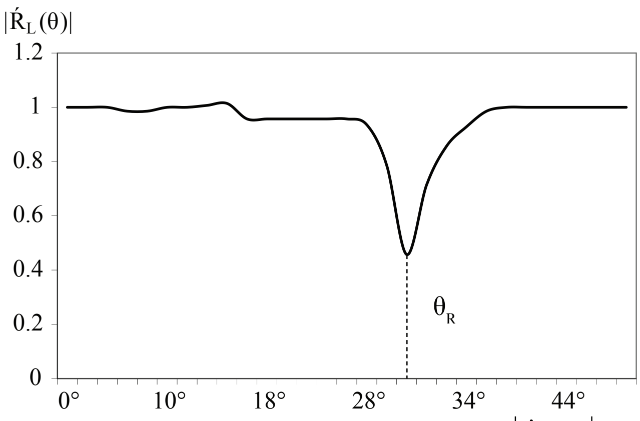

Fig. 1. Ultrasonic wave reflection factor $\left|\dot{R}_L(\theta)\right|$ as a function of refractive angle θ for steel St18GS (immersion UCR-test, thickness of plate)

With steel sheets 0.25<v<0.3 and the speed of longitudinal ultrasonic waves C_L [mm/μs] varies in the interval: $5.7 < C_L < 6.1$.

If in (7) we substitute the modulus of volume elasticity $K_0 = \rho C_L^2 \left(1+\nu\right)/\left(1-\nu\right)$, after some simple calculations we get $E_0 = \rho C_L^2$ for the relation:

$$\frac{E_0^{max}}{E_0^{min}} = \left\{ \frac{0.5 + \left[\left(1 - \nu_{max}\right)/\left(0.5 - \nu_{max}\right)\right]^{3/2}}{0.5 + \left[\left(1 - \nu_{min}\right)/\left(0.5 - \nu_{min}\right)\right]^{3/2}} \right\}^{2/3} \tag{11}$$

Here $E_0 = C_{11}$ is equivalent (effective) modulus of longitudinal elasticity of solid body.

For v_{min} = 0.25 and v_{max} = 0.3 we get from equation (11) the ultimate value of the relation:

$$\left(C_L^{max} / C_L^{min}\right)_{theor} = \sqrt{E_0^{max} / E_0^{min}} = 1.0735$$

and as a result of the experimental data follows that: $\left(C_L^{max} / C_L^{min}\right)_{exp} = 1.0702$.

The coincidence of the theoretical and experimental results confirms the adequacy of the model and proves its application for the needs of ultrasonic non-destructive control of anisotropy in thin-plate construction elements.

Conclusions

Ultrasonic testing with UCR methods of anisotropic thin-plate elements (Imamura, 2003.) allows us to

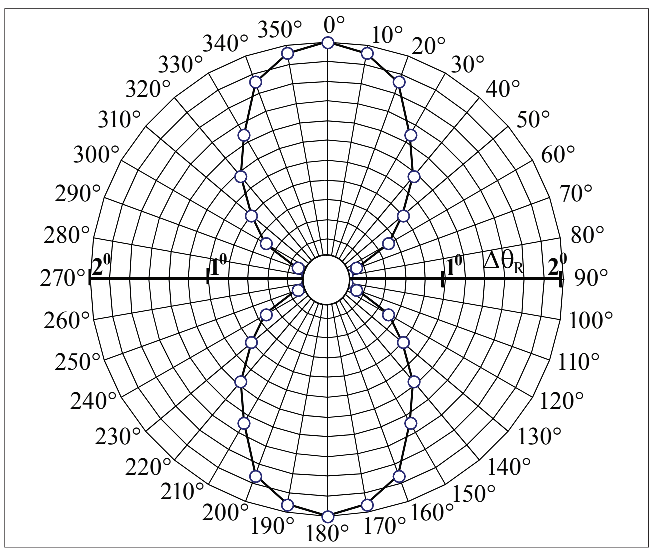

Fig. 2. Directional angle decomposition of θ_R (φ) for thin-plate steel element with mechanical anisotropy

determine the biaxial anisotropy of Poisson's ratio $\nu(\varphi)$. The obtained relation in (Lewi, 2010), who is characteristic for polycrystalline aggregates (FCC or BCC type of crystal structure), gives the possibility to interpolate the values of $G(\varphi)$ and $E_0(\varphi)$ from the experimental data. From the received formulas, we can calculate the value of the index of anisotropy A_c of thin-plate structural elements in agricultural machinery.

References

Bunge, H.-J., 1993. Texture Analysis in Materials Science – Matematical Methods. Göttingen, *Cuvillier Verlag.*

Hill, T. L., 1986. An Introduction to Statistical Thermodynamics. New York, *Dover Publ.*

Hoffmann, H.–J., 2004. On Lidemann's Melting Criterion. *Materials Science and Egineering,* **35** (2): 79-81.

Imamura, T., 2003. Measuring method for the mechanical anisotropy of solids by water immersion ultrasonic sing-around method. Proc.WCU 7-10.09.2003, Paris, pp. 769-772.

Jones, W. and N. March, 1973. Theoretical Solid State Physics. New York, *J. Wiley Inc.*

Kocks, U. F., C. N. Tome and H.–R. Wenks (eds.), 1998. Texture and Anisotropy. Cambridge, *Cambridge University Press*.

Lewi, A., 1988. Ultrasonic refractometric testing of the mechanical anisotropy of thin elements of construction. I Conf. of Non-Destructive Testing 26-27.09.1988, Varna, pp. 405-408 (Bg).

Lewi, A., 2010. Ultrasonic non-destructive testing of the mechanical properties of thin elements of construction. Warsaw, *IFTR Reports*, № 3: 1-184 (Pl).

Randle, V. and O. Engler, 2000. Introduction to Texture Analysis: Macro texture. Microtexture and Orientation Mapping. Washington DC, *Taylor and Francis Publ.*

Rychlewski, J., 1984. On Hooke's law. *Prikl. Math. Mekhan*, **48** (3): 303 – 314 (Ru).

Śliwinski, A., 2000. Ultrasound and its applications. Warsaw, *WNT*, (Pl).

Ushio, M., A. Sadek and M. Morsy, 1993. Welding of Thin Steel Plates Using Submerged Arc Welding. Trans. *JWRI,* **22** (2): 219-225.

Voight, W., 1928. Lehrbuch der kristallphysik (mit ausschluss der kristalloptik). Leipzig, Teubner, *Verlag.*

Received August, 2, 2012; accepted for printing March, 2, 2013.