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MATHEMATICAL MODEL OF DRYING PROCESS AND SUBSEQUENT COOLED SEEDS IN DRYING-COOLING UNIT

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Abstract

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The investigation resulted in the mathematical model of drying process and subsequent cooling of seeds in the drying-cooling unit in a system of nonlinear differential equations in ordinary and partial derivatives. The system of differential equations is reduced to a system of integral equations and transfer functions. An integral equation describes the distribution of temperature of drying agent in the office in time. The results are applied to the synthesis system to stabilize process parameters of heat treatment of seeds in the drying-cooling unit.

Key words: seeds, drying, cooling, drying, cooling unit, a mathematical model, the transfer function

Introduction

Stochastic features of the drying process due to random interaction phases or random nature of the geometry of the boundary conditions in an oven. Constant increase in performance requirements of quality management systems drying process requires a complexity of their mathematical models, taking into account the nonlinearities and random perturbations, in connection with which we come to the synthesis of nonlinear control for stochastic systems.

In solving this problem is important to develop approximate methods of synthesis, comparable in complexity to the methods of synthesis of linear systems.

Typically, the study of nonlinear systems uses linearization (the transition from nonlinear functional operator of a dynamical system to a linear differential operator in the form of the Fokker-Planck-Kolmogorov (FPK)). FPK equation is always true in describing the behavior of the system near its steady state. The results are applied to the synthesis system to stabilize process parameters of heat treatment of seeds in the drying-cooling unit.

Materials and Methods

Let the drying process, fuctioning in a random noise, characterized by n variables $x_1, x_2, \dots, x_n$. Regarding the variables $x_1, x_2, \dots, x_n$, called principal coordinates, the technological process can be regarded as a stochastic dynamic system, the equation of state which has the form (Krasovskii, 1974.):

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$$\dot{x}_i + F_i(x_1, x_2, \dots, x_n) = \xi_i(t) \quad (i = 1, 2, \dots, n) \quad (1)$$

where $\xi_1(t)$, $\xi_2(t)$, … , $\xi_n(t)$ - random functions of time such as Gaussian white noise with a matrix of spectral densities

$S = \|S_{ik}\|, \quad S_k = const$ and correlation matrix $R(\tau) = \|R_{ik}(\tau)\| = \|S_{ik}\,\delta(\tau)\|, \quad \delta(\tau) -$ delta function.

The equation for free motion (the system without interference) is often used in the theory of dynamical systems:

$$\dot{x}_i + F_i(x_1, x_2, \dots, x_n) = 0 \quad (2)$$

Complete characteristic of a random process with the components $x_1(t), x_2(t), \dots, x_n(t)$, is the density of the n - dimensional probability distribution $p = p(x_1, x_2, \dots, x_n)$, where t -time.

The functions p p and p_0 must satisfy the normalization condition:

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\dots\int_{-\infty}^{+\infty} p(x_1, x_2, \dots, x_n, t)\,dx_1 \dots dx_n = 1 \quad (3)$$

The set of phase trajectories of system (2), corresponding to a set of random initial coordinate values of the density probability distribution $p_0 = p(x_1, x_2, \dots, x_n, t_0)$, we consider the trajectories of particles of a «gas» in the phase space. Density of «gas», average number of particles per unit volume of phase space is a probability density $p(x_1, x_2, \dots, x_n, t)$

Isolate the phase space of the elementary rectangular parallelepiped volume. $dx_1 dx_2 \dots dx_n$

Invoking equation (2), we find that the difference in the average number of particles, the coming and going through the brink of a dedicated box for the time dt, divided by this interval of time dt, is:

$$\sum_{i=1}^{n}[(x_1, \dots, x_i, \dots, x_n, t)\dot{x}_i(x_1, \dots, x_i, \dots, x_n) - p(x_1, \dots, x_i + dx_i, \dots, x_n, t) \times$$

$$\times \dot{x}_i(x_1, \dots, x_i + dx_i, \dots, x_n)]\, dx_1 \dots dx_{i-1}\, dx_{i+1} \dots dx_n =$$

$$= dx_1\, dx_2 \dots dx_n \sum_{i=1}^{n} \frac{\partial}{\partial x_i}(pF_i)$$

Get the FPK equation for the system without noise (2):

$$\frac{\partial p}{\partial t} = \sum_{i=1}^{n} \frac{\partial}{\partial x_i}(pF_i) \quad (4)$$

For system (1) exposed to Gaussian white noise $\xi_1(t)$, $\xi_2(t)$…, $\xi_n(t)$ with the mutual spectral densities S_{ik} *(i, k = 1, 2, …, n),* the FPK equation has the form /1, 2/:

$$\frac{\partial p}{\partial t} = \sum_{i=1}^{n} \frac{\partial}{\partial x_i}(pF_i) + \frac{1}{2}\sum_{k=1}^{n} S_{ik} \frac{\partial^2 p}{\partial x_i \partial x_k} \quad (5)$$

Situation is somewhat different from the issues of synthesis of dynamic systems. It is shown that by analyzing the changes in the probability density in phase space can obtain an approximate solution of some optimal control (Evlanov and Konstantinov, 1976).

Results and Discussion

Consider a unit for drying and cooling the seed of the technological process is presented in Figure 1. Seeds with initial moisture w_0 and the initial temperature θ^0 using the boot device enters the dryer drum. The dried seeds from the finite temperature θ and the final moisture content w unloaded from the drum and fed to the cooling chamber, where the seeds are transported to the exit directional flow of coolant. Chilled to the final temperature θ_2 and brought to a final moisture content of w_2, seeds are fed to storage. To reduce the exergetic cost of the drying process in the combined unit warm outgoing air after cooler is fed into the chamber bias.

In describing the process thermal treatment of seeds in the drying-cooling unit will adopt the following notation: a_0 - specific surface of the seeds, m^2/m^3; w, w_0 - the final and initial seed moisture kg/kg; C_A, C_S - the specific heat capacity of the drying agent and seeds, kkal/kg grad; F - surface heat transfer between fluidized bed and the wall unit, m^2; G_S, F_A - volume flow rate of seeds and the drying agent, m^3/h; ρ_A, ρ_S - the density of the drying agent and seeds, kg/m^3; l - the spatial coordinate,

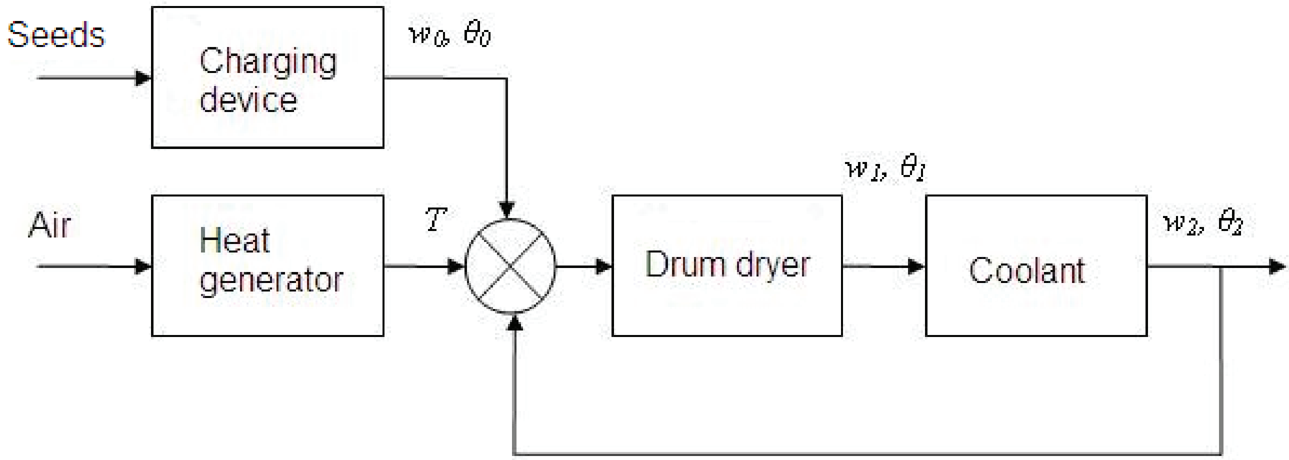

Fig. 1. Scheme of technological process for drying and subsequent cooling of seeds in the combined unit

m; L - length of the drying drum, m; S - cross-sectional area of the drum, m^2; V_D - volume of the drum, m^3; t - time, h; T - temperature of drying agent (coolant), grad; T_{DW} - temperature of the outer drum walls, grad; ω_A, ω_S - linear speed drying agent and seeds, m/h; α - heat transfer coefficient between the drying agent and the seeds, kkal/m^2 h grad; α_{CT} - heat transfer coefficient between fluidized bed and the wall cooler, kkal/m^2 h grad; θ, θ^0 - the final and initial temperature of the seeds, grad; ε - porosity of the fluidized bed (ε = 0.4÷0.6); λ - coefficient of heat transfer, kkal/m^2 h grad; D - coefficient of transverse mixing.

Heat-balance equation for an elementary volume per unit time with respect to cross-mixing has the form (Lipatov, 1973):

$$\varepsilon\, C_A \rho_A \frac{\partial T}{\partial t} - \omega_A C_A \rho_A \frac{\partial T}{\partial l} = -\alpha\,(1-\varepsilon)(T-\theta) + D\frac{\partial^2 T}{\partial l^2}.$$

Similarly:

$$C_S \rho_S \frac{\partial \theta}{\partial t} + \omega_S \frac{C_S \rho}{1-\varepsilon} \frac{\partial \theta}{\partial l} = \alpha\, \frac{1-\varepsilon}{1-\varepsilon}(T-\theta) + \lambda\, \frac{F\,(\Delta T)}{S\,(1-\varepsilon)}.$$

Taking fluidized bed of solid phase, the object of ideal mixing, and integrating with respect to the parameter l_L, we obtain:

$$c_S \rho_S V_D\,(1-\varepsilon)\frac{d\theta}{dt} = G_S c_S \rho_S \left(\theta^0 - \theta\right) + (1-\varepsilon)\,\alpha S \int_0^L (T-\theta)\,dl + \lambda F L \left(T_{DW} - \theta\right)$$

The material balance equation for the volume element has the following form:

$$S\,(1-\varepsilon)\frac{\partial w}{\partial t} = G_M \frac{\partial w}{\partial l}.$$

Taking the model drum on a solid phase as the object of ideal mixing, we find:

$$\frac{\partial w}{\partial t} = G'_M\,(w_0 - w); \qquad (6)$$

where $G'_S = G_S / [(1-\varepsilon) V_D \rho_S]$

$$\frac{d\theta}{dt} = G'_S\left(\theta^0 - \theta\right) + \alpha F' \left(T_{CT} - \theta\right) + \alpha S' a_0 \int_0^L (T-\theta)\,dl\,; \qquad (7)$$

$$\varepsilon\, C_A \rho_A \frac{\partial T}{\partial t} - \omega_A C_A \rho_A \frac{\partial T}{\partial l} = -\alpha\,(1-\varepsilon)\,a_0\,(T-\theta) + D\frac{\partial^2 T}{\partial l^2}. \qquad (8)$$

Thus, the mathematical model of drying process of seed is a system of nonlinear differential equations in ordinary and partial. We show how this system can be reduced to a system of integral equations and transfer functions.

The integrated process model is as follows:

$$w(t) = \int_{-\infty}^{t} G'_S w_0(\tau) \exp\left[-\int_{\tau}^{t} G'_S\, dr\right] d\tau = w_0 e^{-G'_S (t-\tau)} \qquad (9)$$

is a decreasing exponential function, with *t<0, w(t)=0.*

A dynamic element can be regarded as an aperiodic link with the transfer function of the form:

$$W_1(s)=\frac{k_1}{T_1 s+1}, \quad (10)$$

where k_1 - transfer coefficient, T_1 - time constant.

Rewrite equation (7) as:

$$\frac{d\theta}{dt}=-\xi\theta+u_2(t), \quad (11)$$

$$u_2(t)=G_S'\theta^0(t)+\alpha F'T_{DW}+\alpha S'a_0\int_0^L T(l)\,dl. \quad (12)$$

Now we reduce the equation (11) to integral form:

$$\theta(t)=\int_{-\infty}^{t} K(t-\tau)u_2(\tau)\,d\tau \quad (13)$$

Model the process of changing temperature of the drying agent can be an inertial element with the transfer function of the form:

$$W_2(s)=\frac{k_2}{T_2 s+1}, \quad (14)$$

We obtain the integral equation describing the distribution of temperature of drying agent in the office in time:

$$T(l,t)=\int_0^t \sqrt{\frac{\omega_\Gamma^*}{4\pi D\tau}}\exp\left[-\frac{(t-\tau)^2}{4\pi D^*\tau}\right]\exp\left[-k^*\frac{\tau}{\omega_\Gamma^*}\right]\theta(\tau)\,d\tau \quad (15)$$

Integrating link has a transfer function of the form $W_3(s)=\frac{k}{s}$.

Graphic image of the drying-cooling unit, described in state space is represented in Figure 2. We believe that all state variables of the object known as the output variables. The variables on the output of the model are the initial temperature $\theta^0(t)$ and the humidity of $w_0(t)$ of seeds in the drying chamber, disturbing influences - the final temperature θ and humidity $w(t)$ of seeds, managing influence - the distribution of the coolant temperature $T(l,t)$ in the apparatus and drying time t.

Figure 3 shows the description of the system using the transfer function in the form of two series-connected aperiodic and an integrating stage. The

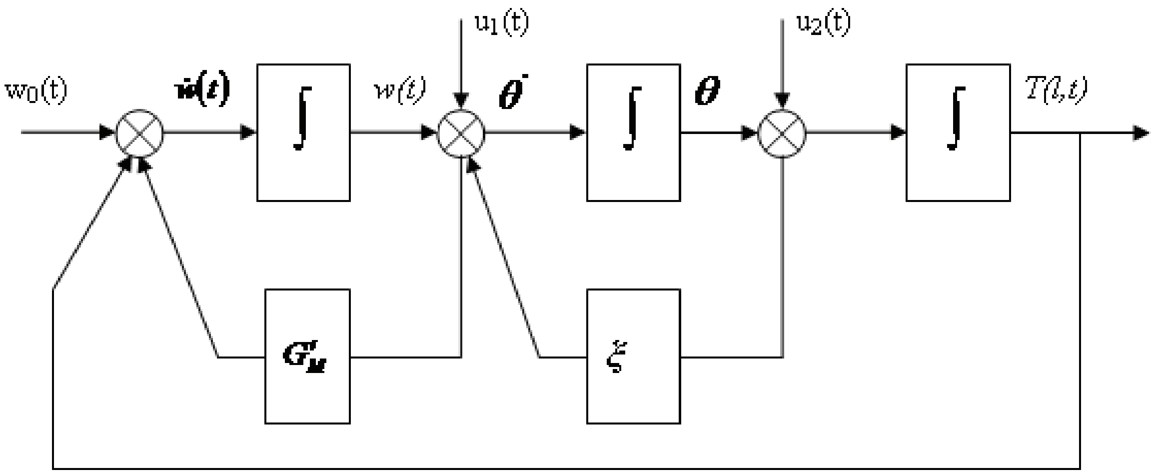

Fig. 2. Block diagram of simulation facility in the state space

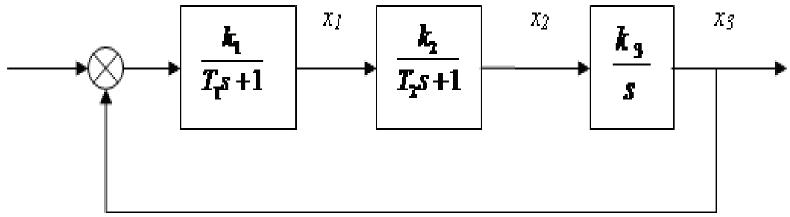

Fig. 3. Block diagram of the object

time constants T_1, T_2 aperiodic links are given or can be identified in the identification system.

Conclusion

Most of the parameters of control and regulation as a result of drying seed must meet the requirements for agro-technical process, and meet certain quality indicators that are associated with the terms of the further storage of seeds and their subsequent processing. Action of the perturbation leads to oscillations of the output parameters of the object that causes the need to optimize the technological process of drying seeds.

The resulting mathematical model of drying process of seed provided in the form of a system of nonlinear differential equations in ordinary and partial, and reduced to a system of integral equations and transfer functions, will allow to optimize the drying process of seed and the subsequent development of computer models of the drying process of seed drying and a cooling unit.

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