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Using Cubic Spline Interpolation to Estimate Vertical Soil Water Profile

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Abstract

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The modelling of the soil water flow under the various affecting factors such as meteorological conditions, irrigation, etc. is one of the main parts of the simulation model of the plant productivity. As an input for the model detailed measured data for soil water profile are required. The needed information is not fully available in the operational bulletins or handbooks of agricultural hydrology. In this paper cubic splines are employed for estimation and completion of the missing data and construction of the soil water profile up to 2 m depth. A procedure based on spline interpolation was developed and its applicability was tested with experimental data carried out for two crops: wheat and maize in 4-years period. The estimated profiles were compared to the measured ones. The results obtained for both crops exhibit sufficient correlation.
Key words: soil water, agro hydrology, spline interpolation, simulation models

Introduction

Study on future climatic conditions of Bulgaria shows a trend towards permanent warming and drought. The productivity processes of agricultural crops under forecasting agro meteorological conditions can be simulated using mathematical models. On the base of the results for expectåd yields an assessment of the opportunities for providing of the population during the present century can be made.

The modelling of the soil water flow under the various affecting factors such as meteorological conditions, irrigation, etc. is one of the main parts of the simulation model of the plant productivity. As an input for the model detailed measured data for soil water profile are required. The full needed information is not presently available in the literature. Usually in the operational bulletins records for available soil

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water within soil layers: 0-10 cm, 0-50 cm, 0-100 cm, and 0-200 cm are received. Similar data only for 0-50 cm and 0-100 cm soil layers are published in the handbooks of agricultural hydrology. In most cases the soil water records within 10-cm. or 20-cm. soil layers up to the depth desired are required. Taking into consideration, that the determination of the available soil water within 0-200 cm soil layer is related to the normal plant growing and yield formation, for computer modeling purposes it is needed to estimate the values for soil water at least up to 2-meter depth.

The purpose of this study is development of a procedure for estimation of detailed soil water profile records up to 2-meter depth. Cubic spline interpolation has been used for the completion and restoration the missing data. Data obtained from field experiments carried out in the agro meteorological station Knezha situated on typical chernozem for crops wheat and maize in 4-years period was used to verify the procedure. The profiles estimated were compared to the measured ones. The results obtained for both crops show sufficient correlation. An analytic equation for calculation of available soil water up to 2-meter depth is evaluated.

Material and Methods

Assume that $W_{0\text{-}10i,}$ where i = 1, 2… is the soil water (mm) within 0-10i cm soil layers up to 2-meter soil depth. The soil water (mm) of 10k – 10l cm soil layers is calculated as:

$$W_{10k\text{-}10l} = W_{0\text{-}10l} - W_{0\text{-}10k}$$
$$k = 0, 1, 2\ldots, l = k+1 = 1, 2\ldots \qquad (1)$$

The problem is to estimate a set of discrete data values of soil water $W_{10k\text{-}10l}$ within 0-200 cm soil layer fit to the measured data of soil water $W_{0\text{-}10i}$ within certain soil layers.

To solve the problem one have to first estimate the unknown values of the soil water $W_{0\text{-}10i}$ to the depth desired and then applying the equation (1) - to estimate the data sequence $W_{10\text{-}20}$, $W_{20\text{-}30}$,……$W_{190\text{-}200}$.

The cubic spline interpolation was selected as an appropriate method for estimating the intermediate values between the measured ones. Interpolation with cubic splines means that the vertical soil water profile between the experimental data points can be represented as a set of third-ordered polynomials, defined inside of the closed intervals $[x_{j-1}, x_j]$:

$$W_j(x) = a_{1j} + a_{2j}\,x + a_{3j}\,x^2 + a_{4j}x^3$$
$$j=1, 2\ldots m \qquad (2)$$

with x -the soil depth [cm],

a_{1j}……….a_{4j} - coefficients of third-ordered polynomials, m -number of the experimental data points. As $W_o(0) = 0$, m is also equal to the number of the intervals. To derive interpolating cubic splines, the coefficients of each third-ordered polynomial are evaluated in respect to the following constraints (De Boor, 1978). The polynomials values in the junctions of the intervals must be equal to the measured data value of soil water:

$$\begin{aligned} W_j(x_k) &= W_{0-10k} \qquad j = 1,\ldots\ldots\ldots m \\ W_{j+1}(x_k) &= W_{0-10k} \qquad k = 1,\ldots\ldots\ldots m \end{aligned} \qquad (3)$$

also the first and second derivates at this points must be equal:

$$W_j''(x_k)=W_j'(x_k) \qquad j=1,........m$$
$$W_j''(x_k)=W_j'(x_k) \qquad k=1,........m-1 \tag{4}$$

and for natural cubic spline the second derivates at the end points must be equal to zero:

$$W_1''(0)=0$$
$$W_m''(x_m)=0 \tag{5}$$

The conditions (3), (4), (5) represent a system of 4m-equations with 4m unknowns – the cubic polynomials coefficients. Solving this system for the actual data, a set of third-ordered polynomials is defined that can be employed to compute the missing values of the soil water in the soil layers up to a depth desired.

By the reason of the character of the solved problem - estimating of available water in the soil profile – an additional restriction is imposed on the values estimated:

$$W_{0-10k} \geq 0 \qquad i=1,.........m \tag{6}$$

The corresponding computer program was developed and computer experiments were carried out to test the applicability of the procedure. Data obtained from field experiments in the agro meteorological station Knezha situated on typical chernozem for two crops: wheat and maize in 4-years period were used.

Results and Discussion

The measured data for soil water within 0-10 cm, 0-50 cm, 0-100 cm, and 0-200 cm soil layers was used to calculate the values of the soil water of 10-cm layers up to 1-m depth and the values of the soil water of 20-cm layers within the 100-200 cm soil layer for every month of the year. The soil water of 20-cm layer is calculated after the equation (1) for l =k+2.

The estimated and measured vertical soil water profiles from March to June 2002 and profiles for both crops are plotted together and compared (Figure 1). As seen, the profiles estimated using spline interpolation are smooth and fit well to the actual profiles. According Bojko et al. (1986) the input data smoothing is important for successful numerical calculations.

For further testing of the procedure adequacy, the estimated versus measured values of soil water of 10-cm layers within the 0-100 cm soil layer and of 20-cm layers within the 100-200 cm soil layer for 1-year period were plotted and compared. Linear regression was used to calculate a trend line for the plot. Figure 2 shows the plots of the measured è estimated data for both crops wheat and maize and for every one of 4-years of the field experiment. No significant deviation between the measured è estimated values is available. The procedure using spline interpolation match the data with the correlation coefficient greater than 0.8 (Table 1).

The obtained deviationes not exceeded 10-12 % between estimated and measured data in most cases were the reason on the base of spline interpolation to find out analytic relationships, which can be implemented for assessment of available soil water with sufficient accuracy.

As the measurements of the soil water in the agro meteorological stations in Bulgaria more often are carried out in the layers: 0-50 cm, 0-100 cm and 0-150 cm, it could be useful for practical purposes the determination of the soil water within 0-200 cm soil layer if the data of the upper layers are available.

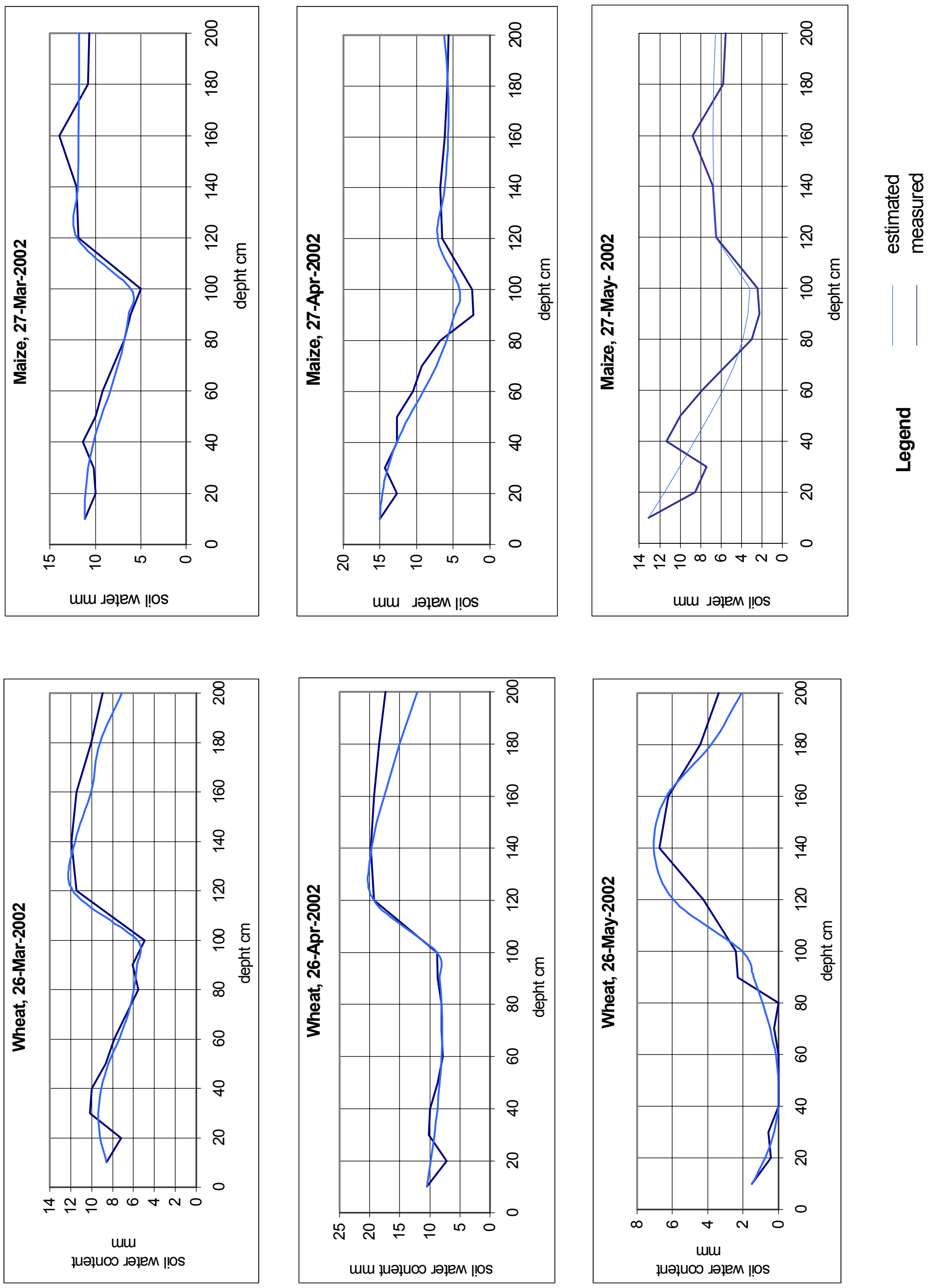

Fig. 1. Measured soil water profiles approximated by cubic spline interpolation

Table 1
Correlation coefficients between calculated and measured values of soil water

Year	Correlation coefficient R^2	
	Maize	Wheat
1999	0.9776	0.7997
2000	0.9902	0.9083
2001	0.9822	0.9489
2002	0.9497	0.8975

To solve the problem the cubic splines ($W_i(x)$, i=1.4) fitted to the data for the soil water within 0-50 cm, 0-100 cm, 0-150 cm and 0-200 cm soil layers $W_{0\text{-}50}$, $W_{0\text{-}100}$, $W_{0\text{-}150}$, $W_{0\text{-}200}$ were developed according to the algorithm suggested by Chapa et al. (1988). The third order polynomial for the interval $150 \le x \le 200\ cm$ is:

$$W_4(x) = a_{14} + a_{24}\,x + a_{34}\,x^2 + a_{44}x^3 \tag{7}$$

with polynomial coefficients $a_{11,}$ a_{12}, a_{13}, a_{14} presented in Table 2.

The equation (7) can be åmployed to compute the unknown soil water value $W_{0\text{-}200}$ within the 0-200 cm layer if x = 200 is substituted into it and the experimental data for soil water within the upper layers $W_{0\text{-}50}$, $W_{0\text{-}100}$, $W_{0\text{-}150}$ are available. The calculations for available water within 0-200 cm soil layer were carried out with the experimental data for maize. The obtained results are shown in Table 3. The devia-

Table 2
Coefficients of the spline interpolation for the interval $150 \le x \le 200\ cm$

Coefficients
$a_{14} = -6.81428571\ W_{0-50} + 27.2571429\ W_{0-100} - 34.6142857\ W_{0-150} + 14.0357143\ W_{0-200}$
$a_{24} = 0.10071428\ W_{0-50} - 0.40285714\ W_{0-100} + 0.55071429\ W_{0-150} - 0.23178571\ W_{0-200}$
$a_{34} = -0.00005143\ W_{0-50} + 0.00102857\ W_{0-100} - 0.00291429\ W_{0-150} + 0.00128571\ W_{0-200}$
$a_{44} = 0.00000086\ W_{0-50} - 0.00000343\ W_{0-100} + 0.00000486\ W_{0-150} - 0.00000214\ W_{0-200}$

Table 3
Measured and calculated values of soil water, relative error

Date	$W_{0\text{-}50}$	$W_{0\text{-}100}$	$W_{0\text{-}150}$	$W_{0\text{-}200}$	$W_{0\text{-}200}$ (calculated)	%
	mm	mm	mm	mm	mm	%
27-Mar-99	53.77	121.9	212.29	297.91	307.66	3.27
27- Apr -99	54.4	121.28	193.25	250.93	265.64	5.86
27- May -99	44.19	98.25	163.6	214.94	231.3	7.61
27-June -99	26.71	71.83	127.88	172.86	186.42	7.84

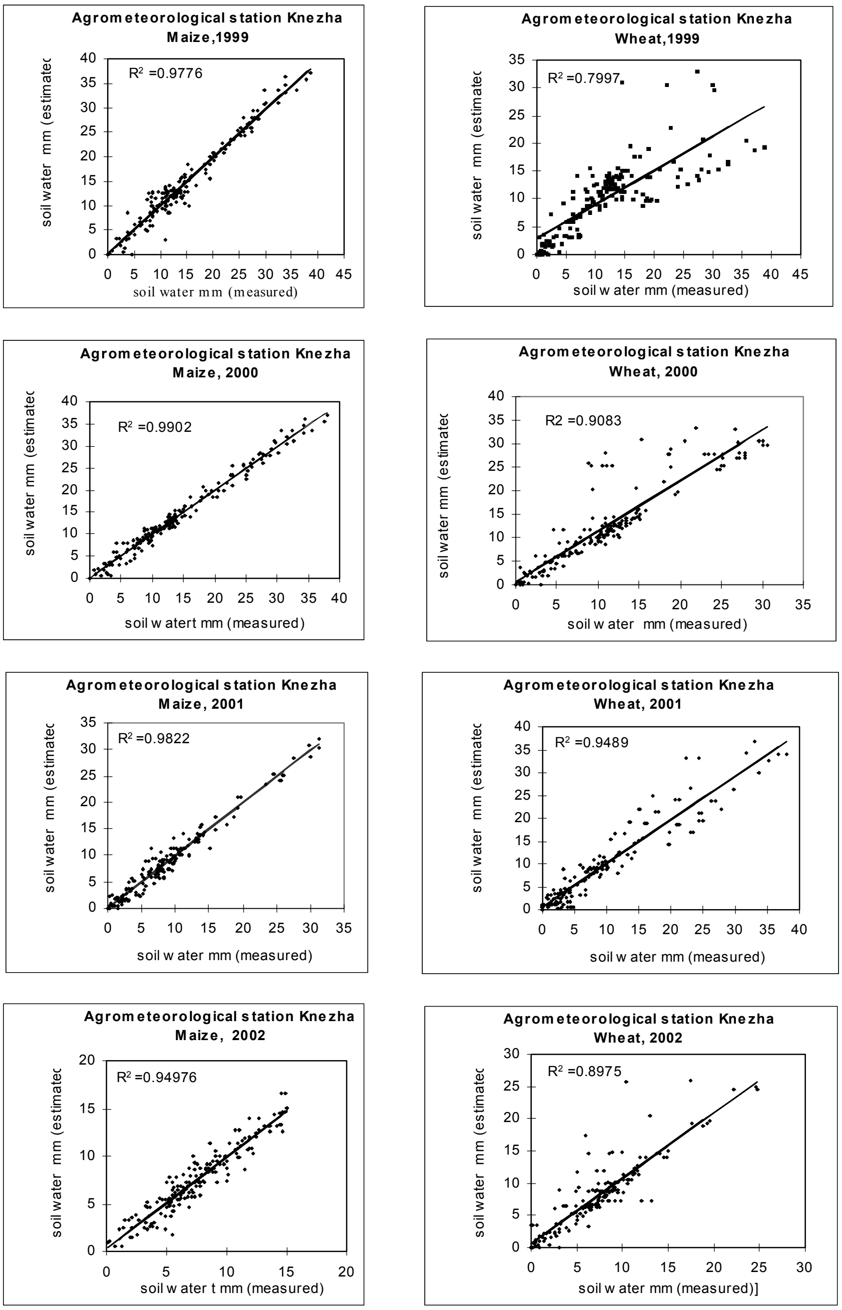

Fig. 2 Results using linear regression to compare measured values versus estimated values of soil water

tion of calculated values from actual ones is 10-15 mm and the relative error is not exceeded 8%.

Conclusions

The results obtained in this study exhibit sufficient correlation between the values of the available soil water calculated using spline interpolation and the actual measured ones in different years and for two crops for the certain soil type. The correlation coefficient ranges from about 0.8 to 0.99. It can be concluded that cubic spline interpolation is an adequate method, which can be implemented for a completion of missing actual data, estimation of the vertical soil water profiles and available soil water assessment.

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